Eur Phys J C **33**, s01, s460–s462 (2004)
Digital Object Identifier (DOI) 10.1140/epjcd/s2003-03-428-2

EPJ C direct
electronic only

Extraction of the x-dependence of the non-perturbative QCD b-quark fragmentation distribution component

E. Ben-Haim[1,2,a]

[1] U. de Paris-Sud, Lab. de l'Accélérateur Linéaire (L.A.L.), Orsay and IN2P3-CNRS
[2] LPNHE, Universités de Paris 6-7 and IN2P3-CNRS

Received: 11 November 2003 / Accepted: 25 November 2003 /
Published Online: 3 December 2003 –

Abstract. Using recent measurements of the b-quark fragmentation distribution obtained in $e^+e^- \to b\bar{b}$ events, registered at the Z pole, the non-perturbative QCD component of the fragmentation distribution has been extracted independently of any hadronic physics modelling. This distribution depends only on the way the perturbative QCD component has been defined. When the perturbative QCD component is taken from a parton shower Monte-Carlo, the non-perturbative QCD component is rather similar with those obtained from the Lund or Bowler models. When the perturbative QCD component is the result of an analytic NLL computation, the non-perturbative QCD component has to be extended in a non-physical region and thus cannot be described by any hadronic modelling. In the two examples, used to characterize these two situations, which are studied at present, it happens that the extracted non-perturbative QCD distribution has the same shape, being simply translated to higher-x values in the second approach, illustrating the ability of the analytic perturbative QCD computation to account for softer gluon radiation than with a parton shower generator.

1 Introduction

Improved determinations of the b-quark fragmentation distribution have been obtained by ALEPH [1], DELPHI [2], OPAL [3] and SLD [4] collaborations which measured the fraction of the beam energy taken by a weakly decaying b-hadron in $e^+e^- \to b\bar{b}$ events registered at, or near, the Z pole.

This distribution is generally viewed as resulting from three components: the primary interaction (e^+e^- annihilation into a $b\bar{b}$ pair in the present study), a perturbative QCD description of gluon emission by the quarks and a non-perturbative QCD component which incorporates all mechanisms at work to bridge the gap between the previous phase and the production of weakly decaying b-mesons. The perturbative QCD component can be obtained using analytic expressions or Monte Carlo generators. The non-perturbative QCD component is usually parametrized phenomenologically via a model.

To compare with experimental results, one must fold both components to evaluate the expected x-dependence:

$$\mathcal{D}_{predicted}(x) = \int_0^1 \mathcal{D}_{pert.}(z) \times \mathcal{D}^{model}_{non-pert.}(\frac{x}{z})\frac{dz}{z} \qquad (1)$$

In the present analysis, $x = \frac{\sqrt{x_E^2 - x_{min}^2}}{\sqrt{1-x_{min}^2}}$ where $x_E = \frac{2E_B}{\sqrt{s}}$ is the fraction of the beam energy taken by the weakly decaying hadron and $x_{min} = \frac{2m_B}{\sqrt{s}}$ is its minimal value. The final and the perturbative components are defined over the $[0, 1]$ interval. As explained, in the following, the non-perturbative distribution must be evaluated for $x > 1$, if the perturbative component is non-physical. The parameters of the model are then fitted by comparing the measured and predicted x-dependence of the b-quark fragmentation distribution. Such comparisons have already been made by the different experiments using, for the perturbative component, expectations from generators such as the JETSET or HERWIG parton shower Monte-Carlo. It has been shown, with present measurement accuracy, that most of existing models, for the non-perturbative part, are unable to give a reasonable fit to the data [1,2,3,4]. Best results have been obtained with the Lund and Bowler models [12,13]. In the following, a method is presented to extract the non-perturbative QCD component of the fragmentation function directly from data, independently of any hadronic model assumption.

[a] Supported by EEC RTN contract HPRN-CT-00292-2002.

2 Extracting the x-dependence of the non-perturbative QCD component

The method is based on the use of the Mellin transformation which is appropriate when dealing with integral equations as given in (1). The Mellin transformation of the expression for $\mathcal{D}(x)$ is:

$$\tilde{\mathcal{D}}(N) = \int_0^\infty dx\ x^{N-1}\ \mathcal{D}(x) \qquad (2)$$

where N is a complex variable. For integer values of $N \geq 2$, the values of $\tilde{\mathcal{D}}(N)$ correspond to the moments of the initial x distribution [1]. For physical processes, x is restricted to be within the $[0,1]$ interval. The interest in using Mellin transformed expressions is that (1) becomes a simple product:

$$\tilde{\mathcal{D}}(N) = \tilde{\mathcal{D}}_{pert.}(N) \times \tilde{\mathcal{D}}_{non-pert.}(N) \qquad (3)$$

Having computed, in the N-space, distributions of the measured and perturbative QCD components, the non-perturbative distribution, $\tilde{\mathcal{D}}_{non-pert.}(N)$ is obtained from (3). Applying the inverse Mellin transformation on this distribution one gets $\mathcal{D}_{non-pert.}(x)$ without any need for a model input:

$$\mathcal{D}_{non-pert.}(x) = \frac{1}{2\pi i} \oint dN\ \frac{\tilde{\mathcal{D}}_{meas.}(N)}{\tilde{\mathcal{D}}_{pert.}(N)}\ x^{-N} \qquad (4)$$

in which the integral runs over a contour in the complex N-plane.

A detailed description of the extraction method can be found elsewhere [5].

3 x-dependence measurement of the non-perturbative QCD component

The method has been applied using three different approaches to evaluate the perturbative QCD component: a parton shower Monte-Carlo (JETSET 7.3 [6]), an analytic NLL computation [7] and another analytic NLL computation using Dressed Gluon Exponantiation (DGE) [8]. The extracted non-perturbative QCD components are presented in Fig. 1. The results corresponding to the NLL+DGE approach are still preliminary and therefore in the following we will concentrate on the two other cases.

For those three perturbative approaches, the extracted non-perturbative QCD component has been found to be compatible with zero for $x < 0.7$. This means that the gluon radiation is well accounted, in this region, by all of them.

When the order of perturbative QCD becomes higher, the non-perturbative part starts at a larger x value. This fact illustrats the ability of higher order QCD computations to account for softer gluon radiation. For the non-perturbative functions, corresponding to JETSET and to the NLL computation of [7] it happens that the non-perturbative QCD component has a similar shape, being simply translated to higher-x values in the case of the analytic NLL QCD.

When the perturbative QCD component is taken from the analytic result of [7] it has been found that, because of the analytic behaviour of the perturbative QCD component, the non-perturbative QCD distribution must be

[1] By definition $\tilde{\mathcal{D}}(1)$ $(= 1)$ corresponds to the normalization of $\mathcal{D}(x)$.

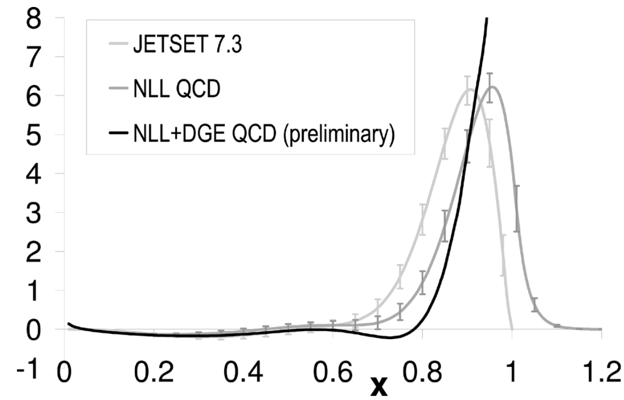

Fig. 1. The extracted non perturbative QCD component for the three perturbative approaches. The result corresponding to NLL+DGE has not been yet calculated for $x > 0.95$

extended above $x = 1$. This has not a physical meaning. It is directly related to the breakdown of the NLL QCD approach when x gets close to 1, and is necessary in order to compensate for the unphysical behaviour of the perturbative QCD component in this region. The x-behaviour of the non-perturbative component, for $x > 1$, is determined by the possible existence of a zero in $\tilde{\mathcal{D}}_{pert.}(N)$, for $N > 0$.

As the non-perturbative QCD distribution is evaluated for any given value of the x-variable it can be verified if it remains physical over the interval $[0,1]$ when used with a Monte-Carlo generator which provides the perturbative component. The evidence for unphysical regions would indicate that the simulation or the measurements are incorrect. There is not such an evidence in the present analysis.

4 Comparison with models

In Fig. 2, the directly extracted non-perturbative components are compared with distributions taken from models [9,10,11,12,13] whose parameters have been fitted on data from [1]. Results have been obtained by comparing, in each bin, the measured bin content with the integral, over the bin, of the folded expression for $\mathcal{D}_{predicted}(x)$. The results corresponding to the perturbative component from JETSET are summarized in Table 1. When the perturbative QCD component is taken from JETSET, the non-perturbative QCD component is rather similar in shape with those obtained from the Lund symmetric [12] or Bowler [13] models, and is rather different from distributions obtained in other models that have tails at low x (e.g. Peterson [10], Collins-Spiller [11]).

When the perturbative QCD component is taken from the analytic result of [7], the fact that the non-perturbative QCD component extends into a non-physical region implies that it cannot be described by any given physical hadronic model. Therefore the results of the corresponding fits are meaningless, and have not be included in Table 1. A parametrization of the extracted non-perturbative distribution has been proposed in [5].

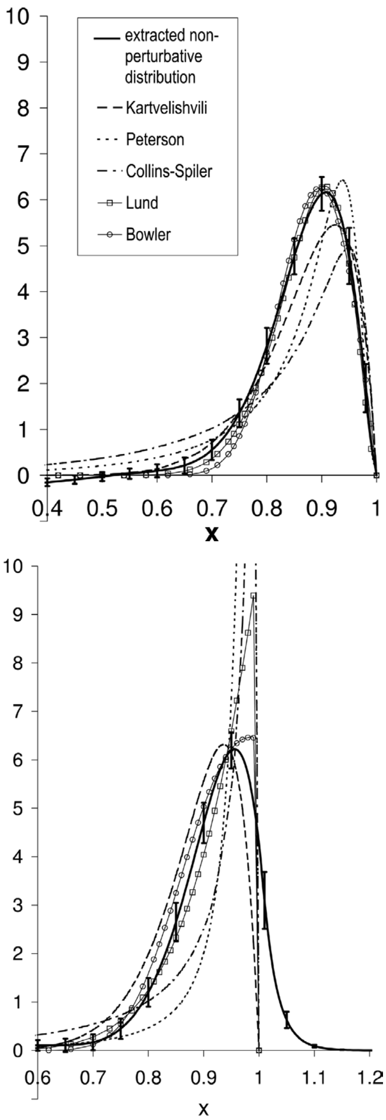

Fig. 2. Comparison between the directly extracted non-perturbative component (thick full line) and the model fits on data taken from [1]. *Left:* the perturbative QCD component is taken from JETSET. *Right:* the theoretical perturbative QCD component [7] is used

Table 1. Values of the parameters and of the χ^2/NDF obtained when fitting results from (1) for different models of the non-perturbative QCD component, to the measured b-fragmentation distribution. The perturbative QCD component has been taken from JETSET. The Lund and Bowler models have been simplified by assuming that the transverse mass of the b-quark, $m_{b\perp}$, is a constant. The last quoted uncertainty corresponds to the variation induced by selecting, in the fitting procedure, between five and nine eigenvalues of the measured error matrix

Model	param.	χ^2/NDF
Kartvelishvili [9] $x^{\epsilon_b}(1-x)$	$\epsilon_b = 12.3 \pm 0.7 \pm 0.4$	35/6
Peterson [10] $\frac{1}{x}\left(1-\frac{1}{x}-\frac{\epsilon_b}{1-x}\right)^{-2}$	$\epsilon_b = (4.1^{+0.4}_{-0.3}{}^{+0.2}_{-0.1}) \times 10^{-3}$	47/6
C.S [11] $\left(\frac{1-x}{x}+\frac{\epsilon_b(2-x)}{1-x}\right)\left(1+x^2\right)\cdot\left(1-\frac{1}{x}-\frac{\epsilon_b}{1-x}\right)^{-2}$	$\epsilon_b = (3.3 \pm 0.5^{+0.4}_{-0.9}) \times 10^{-3}$	117/6
Lund [12] $\frac{1}{x}(1-x)^a \exp\left(-\frac{bm^2_{b\perp}}{x}\right)$	$a = 1.68 \pm 0.18^{+0.09}_{-0.06}$ $bm^2_{b\perp} = 15.6 \pm 1.2^{+0.5}_{-0.4}$	7/5
Bowler [13] $\frac{1}{x^{1+bm^2_{b\perp}}}(1-x)^a\cdot \exp\left(-\frac{bm^2_{b\perp}}{x}\right)$	$a = 0.89 \pm 0.11 \pm 0.10$ $bm^2_{b\perp} = 75. \pm 9.^{+.5}_{-9.}$	18/5

5 Conclusions

The measured b-quark fragmentation distribution has been analysed in terms of its perturbative and non-perturbative QCD components.

The x-dependence of the fragmentation distribution has been extracted in a way which is independent of any model for non-perturbative hadronic physics. It depends closely on the way the perturbative QCD component has been evaluated. The obtained distribution differs markedly from those expected from various models. When the perturbative QCD component is taken from JETSET the extracted distribution is rather similar in shape with those expected from the Lund symmetric [12] or Bowler [13] models. When the perturbative QCD component is taken from the analytic result of [7], the non-perturbative QCD distribution must be extended above $x = 1$.

Consistency checks, on the matching between the measured and predicted b-fragmentation distribution, can be defined which provide information on the determination of the perturbative QCD component itself.

The non-perturbative component, extracted in this way, is expected to be valid in a different environment than e^+e^- annihilation, as long as the perturbative QCD part is evaluated within the same framework (analytic QCD computation or a given Monte Carlo generator), and using the same values for the parameters entering into this evaluation as m_b^{pole}, $\Lambda^{(5)}_{QCD}$ or generator tuned quantities.

References

1. A. Heister et al., ALEPH Collaboration: Phys. Lett. B **512**, 30 (2001)
2. G. Barker et al., DELPHI Collaboration: DELPHI 2002-069 CONF 603
3. G. Abbiendi et al., OPAL Collaboration: Eur.Phys.J. C **29**, N4 463 (2003)
4. K. Abe et al., SLD Collaboration: Phys. Rev. D **65**, 092006 (2002), Erratum-ibid. D **66**, 079905 (2002)
5. E. Ben-Haim, P. Bambade, P. Roudeau, A. Savoy-Navarro, and A. Stocchi: hep-ph/0302157
6. T. Sjöstrand: Comp. Phys. Comm. **82**, 74 (1994)
7. M. Cacciari and S. Catani: Nucl. Phys. B **617**, 253 (2001)
8. M. Cacciari and E. Gardi: Nucl. Phys. B **664**, 299 (2003)
9. V.G. Kartvelishvili, A.K. Likehoded, and V.A. Petrov: Phys. Lett. B **78**, 615 (1978)
10. C. Peterson, D. Schlatter, I. Schmitt, and P.M. Zerwas: Phys. Rev. D **27**, 105 (1983)
11. P. Collins and T. Spiller: J. Phys. G **11**, 1289 (1985)
12. B. Andersson, G. Gustafson, and B. Soderberg: Z. Phys. C **20**, 317 (1983)
13. M.G. Bowler: Z. Phys. C **11**, 169 (1981)